Homework 10 for Columbia B9136

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Exercise 1.

Suppose you are selling one item to T agents whose valuations lie in $\{r_1, \dots, r_m\}$, where $r_1 > \dots > r_m \geq 0$, with the goal of maximizing revenue in a prior-free setting. Compute the following "competitive ratio" of randomized DSIC/DSIR mechanisms:

$$\sup_{\text{DSIC/DSIR } \tilde{X}, \tilde{P}} \inf_{\mathbf{F} \in \Delta(\{r_1, \dots, r_m\})^T} \frac{\mathbb{E}_{\mathbf{V} \sim \mathbf{F}} \left[\sum_{t=1}^{T} \tilde{P}_t(\mathbf{V}) \right]}{\text{ExpectedVirtualSurplus}(\mathbf{F})}.$$

Note that $\mathbf{F}$ is the product of independent valuation distributions, in which case the Expected Virtual Surplus represents the optimal expected revenue (from Myerson's mechanism) when the valuation distributions are known.

Hint: Establishing the competitive ratio requires proving both an upper and lower bound. This question is easier than it looks, as you have already seen the answer in the course (you may reuse results from the course). The outer "sup" should technically allow a *distribution* over randomized DSIC/DSIR mechanisms, but this additional layer is not needed to achieve the optimal competitive ratio.

Exercise 2. Construct an instance of the online pricing problem with a single item, where buyer valuations are drawn from known independent distributions, in which an online pricing policy can earn at most half the revenue of Myerson's mechanism in expectation.

Hint: Convert the 1/2 counterexample from prophet inequalities into a pricing instance.

Exercise 3.

1. Suppose there are two buyers whose valuations are independently and uniformly drawn from {1,3,4,8}. Calculate the expected ironed virtual surplus.

2. Explicitly write out the allocation rule X and payment rule P of Myerson's optimal auction, for every possibility of the realized valuation profile (V_1, V_2). Calculate the expected payment to the seller.

   Hint: You can choose how to tie-break. The answer should equal the expected ironed virtual surplus.

3. Suppose the allocation rule X was forced to be deterministic and "fair" in that if $X_t(\mathbf{V}) = 1$ for some agent $t \in \{1, 2\}$, then $V_t \geq V_{3-t}$ (that is, if some agent wins the item, then they must have a valuation no smaller than the other agent). Prove that the optimal expected revenue from parts 1 and 2 can no longer be attained.