

Homework 6 for Columbia B9136

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Exercise 1. Show that in the assortment setting, FLU (the “CDLP”) is not more powerful than knowing what choices customers would make. More precisely, suppose ϕ_t is defined by a random utility model (equivalently, distribution over lists) for all t , and let $\mathbb{E}[\text{OFF}]$ denote the expected reward achievable when assortments are decided knowing the realizations of all lists in advance. Construct a family of instances for which $\text{FLU}/\mathbb{E}[\text{OFF}]$ approaches 0.

For this question, you are not allowed to restrict the family \mathcal{S} of feasible assortments.

Hint: The counterexample requires only one time step. You may draw inspiration from the upper bound for the “weight-dependent” setting (see Lecture 2).

Exercise 2. Consider the static assortment calendar setting, but suppose that $i_j \neq i_{j'}$ if $j \neq j'$. Put another way, here we can assume that $n = m$ and all products $j \in [n]$ correspond to exactly a single resource $i \in [m]$ that is sold at price r_i . Show that in this special case, the static assortment calendar policy from class has the improved guarantee of

$$1 - e^{-k} \frac{k^k}{k!}$$

relative to FLU, where $k := \min_{i \in [n]} k_i$.

Hint: You may use (without proof) the fact that if D_1, \dots, D_T are independent Bernoulli random variables whose means sum to at most k , then

$$\frac{\mathbb{E}[\min\{\sum_t D_t, k\}]}{\mathbb{E}[\sum_t D_t]} \geq \frac{\mathbb{E}[\min\{\text{Pois}(k), k\}]}{k} = 1 - e^{-k} \frac{k^k}{k!}. \quad (0.1)$$

Note that applying (0.1) is trickier than before, because in online assortment optimization the events of a product being sold are no longer independent across time, being dependent on other products selling out. You have to think carefully about how to overcome this. Make sure your proof uses substitutability, as the result is false otherwise.

Exercise 3. Establish $\sup_{\mathbf{F} \in \Gamma(F_1, \dots, F_T)} \text{OFF}(\mathbf{F}) = \text{FLU}(F_1, \dots, F_T)$ for any distributions F_1, \dots, F_T supported on values $r_1, \dots, r_n \geq 0$.

Hint: Use the LP formulations from class. Given a feasible solution to one LP, you want to construct a feasible solution to the other LP with the same objective value. The hard direction is given a feasible solution to $\text{FLU}(F_1, \dots, F_T)$, how to construct a coupling for $\mathbf{F} \in \Gamma(F_1, \dots, F_T)$.