Homework 7 for Columbia B9136

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Exercise 1. In class we proved that the regret in the Multi-secretary problem is at most $2R/\lambda_{\min}^2$. Prove the stronger result that the regret is at most

$$\frac{2R}{\lambda_{\min}^2} e^{-\lambda_{\min}}.$$

Hint: This does not require any substantially different argument; a tighter analysis of the inequalities from class will do.

Exercise 2. Consider the Network Revenue Management problem (the Multi-secretary problem is a special case). Let $t \in [T]$, $\mathbf{b} \in \mathbb{Z}_{\geq 0}^m$, $X_t \in \{0,1\}$, and suppose that $\mathbf{b} - \mathbf{a}_{\theta_t} X_t \geq 0$ (i.e., we can only set $X_t = 1$ if $\mathbf{b} \geq \mathbf{a}_{\theta_t}$). Show that

$$J_t^{\mathsf{OFF}}(\mathbf{b}) - J_{t+1}^{\mathsf{OFF}}(\mathbf{b} - \mathbf{a}_{\theta_t} X_t) - r_{\theta_t} X_t \leq LR \tag{0.1}$$

where recall that L is an upper bound on $\|\mathbf{a}_{\theta_t}\|_1$, the number of resources required by type θ_t, and that all product revenues are at most R.

Hint: The statement to prove is deterministic—it holds conditioned on any realization of $\theta_t, \ldots, \theta_T$. If $X_t = 0$ then in fact $J_t^{\mathsf{OFF}}(\mathbf{b}) - J_{t+1}^{\mathsf{OFF}}(\mathbf{b}) \leq r_{\theta_t} \leq R$. However if $X_t = 1$ then $J_t^{\mathsf{OFF}}(\mathbf{b}) - J_{t+1}^{\mathsf{OFF}}(\mathbf{b} - \mathbf{a}_{\theta_t}) - r_{\theta_t}$ can be as large as LR.