Homework 8 for Columbia B9136

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due Apr 7, 11:59pm

Exercise 1. In this exercise we consider Pseudo-dimension for the Assortment Optimization and Optimal Stopping problems. For reward maximization problems, you can define the loss to be the negative of the reward, in the definition of pseudo-shattering.

Consider Assortment Optimization with n products, all having the same price.

1. Show that n lists can be pseudo-shattered.
2. Show that it is not possible to pseudo-shatter more than n lists. Therefore, the Psuedo-dimension is exactly n.

Consider Optimal Stopping over T time steps. In class we saw that the class of stopping policies defined by T thresholds $(\tau_t)_{t=1}^T$ that can vary with t but do not depend on observations can shatter T samples.

1. Suppose we restrict the policies so that $\tau_1 = \cdots = \tau_T$. Compute the Pseudo-dimension (this requires both an upper bound and lower bound).
2. Suppose we expand the policies so that thresholds can depend on observations, i.e. $\tau_1(\cdot)$ is a mapping from V_1 (the first valuation observed) to threshold. Show that any positive integer d number of samples can be shattered, i.e. the Pseudo-dimension is ∞.

   Hint: Even $T = 1$ should work.

Exercise 2. Show that Uniform Stability is not satisfied without regularization. In particular, for any positive integer N, prove that

$$\sup_{\xi_1,\ldots,\xi_N,\tilde{\xi}\in[0,1]} \Big(\ell(A(\tilde{\xi},\xi_2,\ldots,\xi_N),\xi_1) - \ell(A(\xi_1,\xi_2,\ldots,\xi_N),\xi_1)\Big) = \max\{b,h\}.$$